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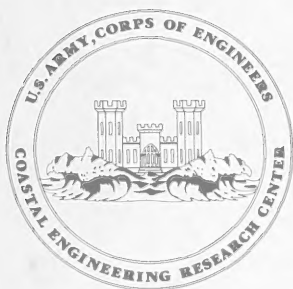
# Irregular Wave Runup on Smooth Slopes

by

John P. Ahrens

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| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number)<br><br>The results of several laboratory studies have been used to develop a method to estimate the wave runup and rundown on plane, smooth slopes caused by irregular wave action. Curves and equations are presented which can be used to compute the 2-percent runup, significant runup, mean runup, and approximate lower limit of rundown. A procedure is suggested for adapting the smooth-slope results to wave runup on rough and porous slopes. Example problems illustrate the use of the material presented. |                       |  |

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## PREFACE

This report presents a method for estimating the magnitude and distribution of wave runup and rundown on plane, smooth slopes caused by irregular wave action. Within the method's range of applicability it supersedes Section 7.212, "Irregular Waves," of the Shore Protection Manual (U.S. Army, Corps of Engineers, Coastal Engineering Research Center, 1977); CETA 77-2 "Prediction of Irregular Wave Runup" by John P. Ahrens; and CETA 78-2 "Revised Wave Runup Curves for Smooth Slopes" by Philip N. Stoa. It also supersedes the parts of CETA 79-1 "Wave Runup on Rough Slopes," by Philip N. Stoa, which estimate wave runup on rough and porous slopes by adjusting the runup for similar wave conditions on smooth slopes using a rough-slope correction factor.

This report was prepared by John P. Ahrens, Oceanographer, under the general supervision of Dr. R.M. Sorensen, Chief, Coastal Processes and Structures Branch, Research Division.

Comments on this publication are invited.

Approved for publication in accordance with Public Law 166, 79th Congress, approved 31 July 1945, as supplemented by Public Law 172, 88th Congress, approved 7 November 1963.

  
TED E. BISHOP  
Colonel, Corps of Engineers  
Commander and Director

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# CONVERSION FACTORS, U.S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U.S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

| Multiply           | by                      | To obtain                               |
|--------------------|-------------------------|---|
| inches             | 25.4                    | millimeters                             |
|                    | 2.54                    | centimeters                             |
| square inches      | 6.452                   | square centimeters                      |
| cubic inches       | 16.39                   | cubic centimeters                       |
| feet               | 30.48                   | centimeters                             |
|                    | 0.3048                  | meters                                  |
| square feet        | 0.0929                  | square meters                           |
| cubic feet         | 0.0283                  | cubic meters                            |
| yards              | 0.9144                  | meters                                  |
| square yards       | 0.836                   | square meters                           |
| cubic yards        | 0.7646                  | cubic meters                            |
| miles              | 1.6093                  | kilometers                              |
| square miles       | 259.0                   | hectares                                |
| knots              | 1.852                   | kilometers per hour                     |
| acres              | 0.4047                  | hectares                                |
| foot-pounds        | 1.3558                  | newton meters                           |
| millibars          | $1.0197 \times 10^{-3}$ | kilograms per square centimeter         |
| ounces             | 28.35                   | grams                                   |
| pounds             | 453.6                   | grams                                   |
|                    | 0.4536                  | kilograms                               |
| ton, long          | 1.0160                  | metric tons                             |
| ton, short         | 0.9072                  | metric tons                             |
| degrees (angle)    | 0.01745                 | radians                                 |
| Fahrenheit degrees | 5/9                     | Celsius degrees or Kelvins <sup>1</sup> |

<sup>1</sup>To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use formula:  $C = (5/9) (F - 32)$ .

To obtain Kelvin (K) readings, use formula:  $K = (5/9) (F - 32) + 273.15$ .

# SYMBOLS AND DEFINITIONS

|           |  |
|-----------|--|
| $d_s$     | water depth at the toe of the slope or structure on which runup occurs   |
| $g$       | acceleration of gravity, 32.2 feet per second squared  |
| $H_s$     | significant wave height at the toe of the structure  |
| $k$       | runup correction factor for scale effects  |
| $L_o$     | deepwater wavelength, $L_o = gT_p^2/2\pi$  |
| $\bar{R}$ | mean runup   |
| $R_s$     | significant runup, i.e., average runup of the highest one-third of wave runups                                       |
| $R_2$     | 2-percent runup, i.e., elevation above the stillwater level exceeded by 2 percent of the runups                      |
| $R_{98}$  | 98-percent rundown, i.e., depth below the stillwater level that is just greater than 98 percent of the rundowns      |
| $r$       | rough-slope runup correction factor, ratio of rough-slope runup to smooth-slope runup, all other conditions the same |
| $T_p$     | period of peak energy density of the wave spectrum   |
| $T_s$     | significant wave period, i.e., average period of the highest one-third of waves                                      |
| $\theta$  | angle formed between the slope of the structure and the horizontal   |
| $\xi$     | surf parameter, $\xi = [(H_s/L_o)^{1/2} \cot \theta]^{-1}$   |



# IRREGULAR WAVE RUNUP ON SMOOTH SLOPES

by  
John P. Ahrens

## I. INTRODUCTION

This report provides guidance on the magnitude and distribution of wave runup and rundown elevations caused by irregular wave conditions similar to those occurring in nature. The results presented are for plane, smooth structures with relatively deep water at the toe of the structure. For these conditions this report supersedes earlier guidance in Section 7.212 of the Shore Protection Manual (SPM) (U.S. Army, Corps of Engineers, Coastal Engineering Research Center, 1977) and Ahrens (1977) which indicate that irregular wave runup has a Rayleigh distribution. Within the range of test conditions this report also supersedes Stoa (1978a) and the parts of Stoa (1979) which estimate wave runup on rough and porous slopes by adjusting the runup on a smooth slope by a correction factor. The range of test conditions covered in this report is discussed in the next section.

## II. IRREGULAR WAVE RUNUP ON PLANE, SMOOTH SLOPES

Three sources of data were used in establishing the methods presented in this report: van Oorschot and d'Angremond (1968), Kamphuis and Mohamed (1978), and Ahrens (1979) which discussed data recently collected at the Coastal Engineering Research Center (CERC). The conditions considered are a structure with a plane, smooth slope fronted by a horizontal bottom offshore. The water depth at the toe of the structure is relatively deep, i.e.,  $3 \leq d_s/H_s \leq 12$ , where  $d_s$  is the water depth and  $H_s$  the significant wave height at the toe of the structure. When there is relatively deep water at the toe of the structure the offshore slope of the bottom has little influence on the wave conditions and therefore little influence on the wave runups. This lack of influence indicates that the runup results presented can be applied to situations where there is an offshore slope. Since the water depth also has little influence on wave runup for conditions when  $d_s/H'_0 > 8$  (Stoa, 1978a), where  $H'_0$  is the deep-water, unrefracted wave height, Stoa's finding suggests that the results of this study should be good for  $d_s/H_s > 12$ .

Three runup parameters were chosen to characterize the runup distribution caused by irregular wave conditions, i.e., the mean runup,  $\bar{R}$ , the significant runup,  $R_s$ , and the 2-percent runup,  $R_2$ . The significant runup is the *average* runup of the highest one-third of wave runups and the 2-percent runup is the elevation *exceeded* by 2 percent of the wave runups.

Figure 1 shows trend-line curves for  $R_2/H_s$ ,  $R_s/H_s$ , and  $\bar{R}/H_s$  for a plane, smooth slope of 1 on 1. These parameters are plotted as a function of the irregular wave steepness parameter,  $H_s/gT_p^2$ , where  $T_p$  is the period of peak energy density of the wave spectrum and  $g$  the acceleration of gravity. The approximate relationship between  $T_p$  and the average period of the significant waves,  $T_s$ , is given by Goda (1974) as

$$T_p \approx 1.05 T_s \quad (1)$$

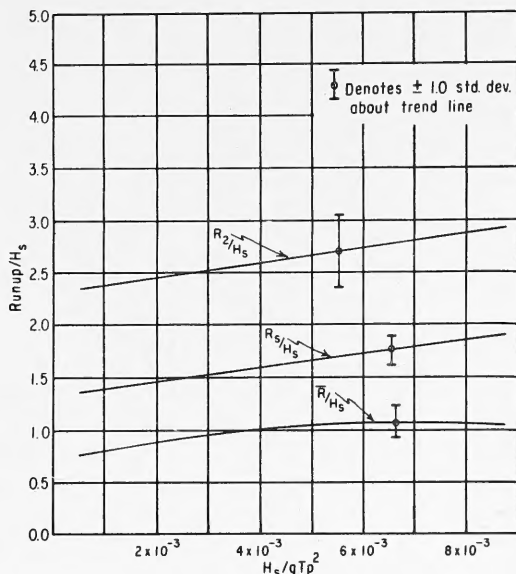


Figure 1. Irregular wave runup parameters versus wave steepness for a plane, smooth slope of 1 on 1,  $d_s/H_s > 3$ .

Figures 2, 3, 4, 5, and 6, which are similar to Figure 1, show trend lines for slopes of 1 on 1.5, 1 on 2, 1 on 2.5, 1 on 3, and 1 on 4, respectively. The trend lines in Figures 1 to 5 are all of the general form

$$\frac{R_x}{H_s} = C_1 + C_2 \frac{H_s}{gT_p^2} + C_3 \left( \frac{H_s}{gT_p^2} \right)^2 \quad (2)$$

where  $R_x$  represents  $R_2$ ,  $R_5$ , or  $\bar{R}$ , and  $C_1$ ,  $C_2$ , and  $C_3$  are dimensionless regression coefficients. In some cases  $C_2$  or  $C_3$  is zero; if  $C_3$  is zero the trend line is straight.

Since a calculator or a computer may be more convenient for calculating the runup parameters than using the figures, Table 1 provides a tabulation of the regression coefficients, along with some statistical parameters which can be used to evaluate how well the curves fit the data. The standard deviation is the standard deviation of the data about the trend-line curves and is shown in Figures 1 to 6 to give an indication of the magnitude of the scatter about the curves. The coefficient of variation is the standard deviation divided by the mean value of  $R_x/H_s$ . Using the coefficient of variation to determine the percent scatter indicates that  $R_5/H_s$  can usually be estimated within the range of  $\pm 5$  to 10 percent about the trend-line curves;  $R_2/H_s$  and  $\bar{R}/H_s$  can be estimated within the range of  $\pm 10$  to 15 percent about the curves.

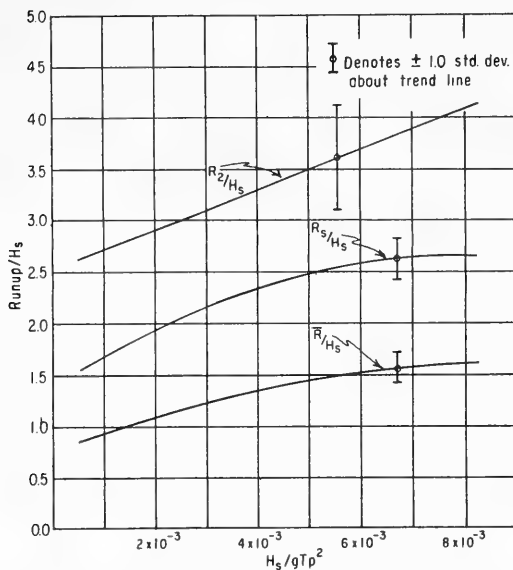


Figure 2. Irregular wave runup parameters versus wave steepness for a plane, smooth slope of 1 on 1.5,  $d_s/H_s > 3$ .

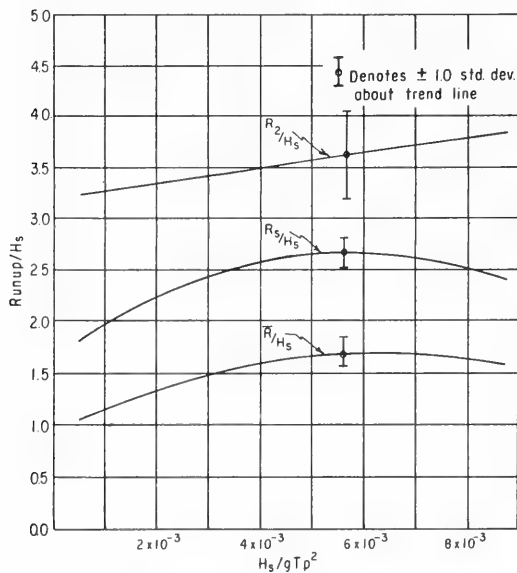


Figure 3. Irregular wave runup parameters versus wave steepness for a plane, smooth slope of 1 on 2,  $d_s/H_s > 3$ .

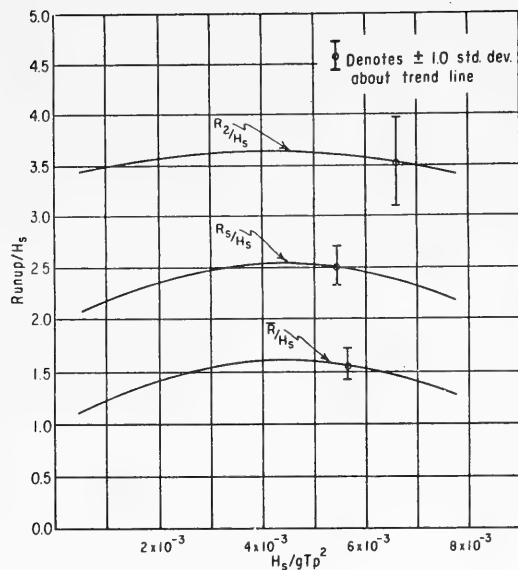


Figure 4. Irregular wave runup parameters versus wave steepness for a plane, smooth slope of 1 on 2.5,  $d_s/H_s > 3$ .

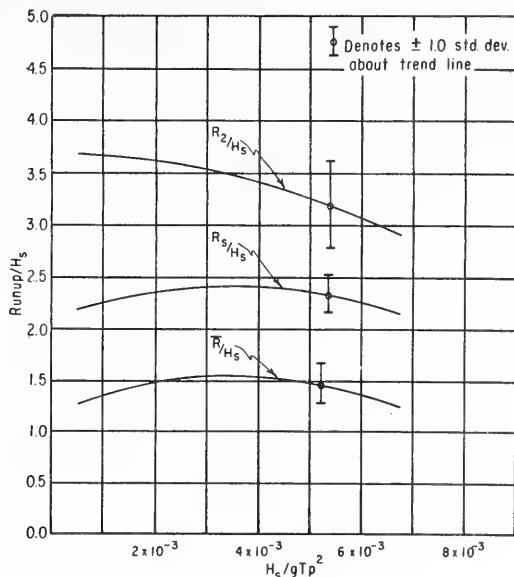


Figure 5. Irregular wave runup parameters versus wave steepness for a plane, smooth slope of 1 on 3,  $d_s/H_s > 3$ .

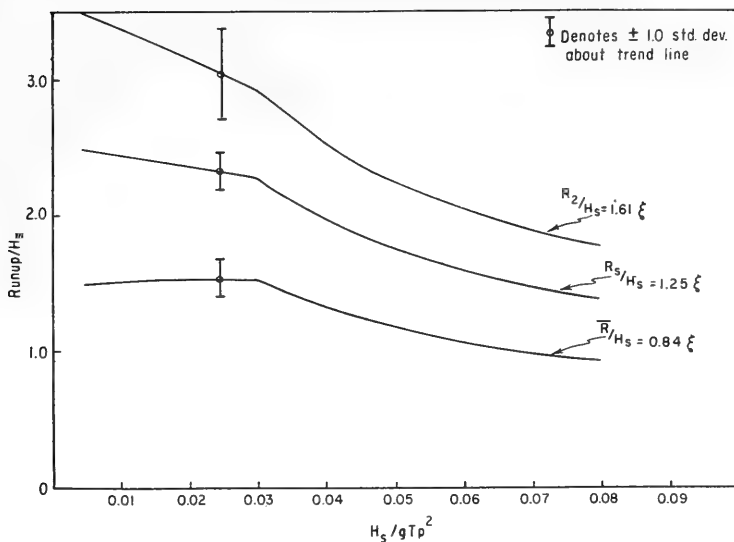


Figure 6. Irregular wave runup parameters versus wave steepness for a plane, smooth slope 1 on 4,  $d_S/H_S > 3$ .

Table 1. Regression coefficients for runup parameters  $R_2/H_S$ ,  $R_S/H_S$ , and  $\bar{R}/H_S$  (see eq. 2).

| Cot $\theta$  | Regression coefficients |                     |                     | Std. dev. | Coeff. of variation |
|---------------|-------------------------|---------------------|---------------------|-----------|---------------------|
|               | $C_1$                   | $C_2$               | $C_3$               |           |                     |
| $R_2/H_S$     |                         |                     |                     |           |                     |
| 1.0           | 2.32                    | $7.15 \times 10^1$  | 0                   | 0.343     | 0.134               |
| 1.5           | 2.52                    | $1.95 \times 10^2$  | 0                   | 0.487     | 0.156               |
| 2.0           | 3.21                    | $7.19 \times 10^1$  | 0                   | 0.421     | 0.123               |
| 2.5           | 3.39                    | $1.29 \times 10^2$  | $-1.61 \times 10^4$ | 0.420     | 0.118               |
| 3.0           | 3.70                    | 0                   | $-1.70 \times 10^4$ | 0.415     | 0.120               |
| 4.0           | 3.60                    | $-2.22 \times 10^2$ | 0                   | 0.330     | 0.117               |
| $R_S/H_S$     |                         |                     |                     |           |                     |
| 1.0           | 1.34                    | $6.61 \times 10^1$  | 0                   | 0.133     | 0.085               |
| 1.5           | 1.38                    | $3.18 \times 10^2$  | $-1.97 \times 10^4$ | 0.195     | 0.094               |
| 2.0           | 1.64                    | $3.57 \times 10^2$  | $-3.09 \times 10^4$ | 0.136     | 0.059               |
| 2.5           | 1.94                    | $2.79 \times 10^2$  | $-3.21 \times 10^4$ | 0.184     | 0.078               |
| 3.0           | 2.11                    | $1.87 \times 10^2$  | $-2.67 \times 10^4$ | 0.190     | 0.081               |
| 4.0           | 2.52                    | $-7.94 \times 10^1$ | 0                   | 0.122     | 0.053               |
| $\bar{R}/H_S$ |                         |                     |                     |           |                     |
| 1.0           | 0.71                    | $1.10 \times 10^2$  | $-8.07 \times 10^3$ | 0.150     | 0.157               |
| 1.5           | 0.75                    | $1.97 \times 10^2$  | $-1.14 \times 10^4$ | 0.143     | 0.119               |
| 2.0           | 0.93                    | $2.42 \times 10^2$  | $-1.93 \times 10^4$ | 0.142     | 0.101               |
| 2.5           | 1.00                    | $2.78 \times 10^2$  | $-3.13 \times 10^4$ | 0.141     | 0.099               |
| 3.0           | 1.19                    | $2.09 \times 10^2$  | $-2.96 \times 10^4$ | 0.181     | 0.123               |
| 4.0           | 1.47                    | $7.25 \times 10^1$  | $-1.70 \times 10^4$ | 0.127     | 0.085               |

Figure 6, for a slope of 1 on 4, is somewhat different than Figures 1 to 5 for steeper slopes. Plunging waves become the dominant breaker type on the 1 on 4 slope, indicating that wave runup can be predicted using a type of formula suggested by Hunt (1959) and used by van Oorschot and d'Angremond (1968). Figure 6 shows trend-line curves, using equation (2), for the less steep wave conditions, i.e.,

$$0.005 \leq \frac{H_S}{gT_p^2} \leq 0.003$$

and a Hunt-type formula is used for the steeper wave conditions, i.e.,  $H_S/gT_p^2 > 0.003$  where plunging waves dominate. The Hunt-type formulas for Figure 6 are given by the equations

$$\frac{R_2}{H_S} = 1.61 \xi \quad (3)$$

$$\frac{R_S}{H_S} = 1.25 \xi \quad (4)$$

$$\frac{\bar{R}}{H_S} = 0.84 \xi \quad (5)$$

where the surf parameter,  $\xi$ , is given by

$$\xi = \frac{1}{(H_S/L_O)^{1/2} \cot \theta} \text{ or } \frac{\tan \theta}{(H_S/L_O)^{1/2}}$$

$L_O$  is the deepwater wavelength given by

$$L_O = \frac{gT_p^2}{2\pi}$$

and  $\cot \theta$  is the cotangent of the angle  $\theta$  between the structure slope and the horizontal.

Figure 7 provides a different perspective and additional insight on the trends to be expected for irregular wave runup. The  $R_S/H_S$  curves from Figures 1 to 6 have been transferred to Figure 7 and plotted versus the surf parameter,  $\xi$ , to show the influence of breaker characteristics on runup. When  $\xi \leq 2.0$ , most of the larger waves in the incident wave train plunge directly on the structure and  $R_S/H_S$  decreases with increasing  $H_S/gT_p^2$  and increasing  $\cot \theta$ . This plunging wave region is where a Hunt-type formula (Hunt, 1959) such as equations (3), (4), and (5) is valid. When  $\xi \geq 3.5$ , no waves plunge on the structure indicating a standing wave condition or surging wave region. The influence of  $H_S/gT_p^2$  and  $\cot \theta$  on  $R_S/H_S$  is reversed for surging waves as

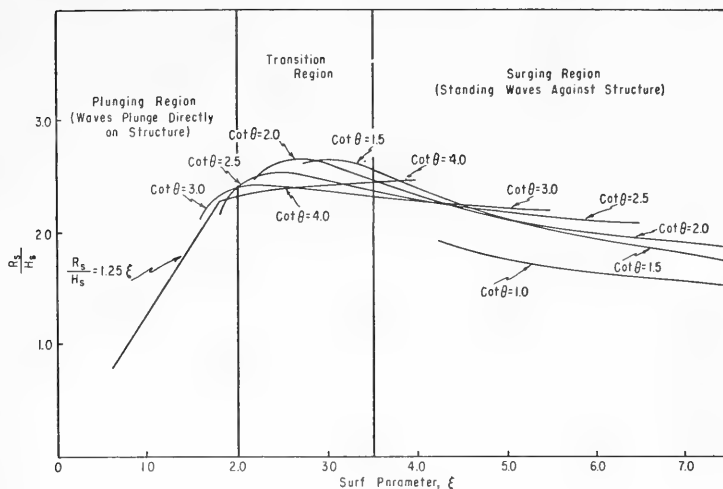


Figure 7.  $R_s/H_s$  versus the surf parameter for  $d_s/H_s > 3$ .

compared to plunging waves; i.e.,  $R_s/H_s$  increases as  $H_s/gT_p^2$  increases and  $\cot \theta$  increases. The reversal of influence creates a transition region,  $2.0 < \xi < 3.5$ , where there is little net influence of  $H_s/gT_p^2$  and  $\cot \theta$  on  $R_s/H_s$ . It is in this transition region that the largest values of  $R_s/H_s$  occur, probably because the most nonlinear surging waves occur in this region. Figure 7 identifies these regions and shows the runup trends. Equations (3), (4), and (5) can be used on slopes flatter than 1 on 4 as long as plunging waves predominate, i.e.,  $\xi \leq 2.0$ .

All the results in this report were obtained in relatively small-scale laboratory studies and must be corrected for scale effects (Stoa, 1978a). The correction for scale effects of wave runup on smooth slopes can be found in Stoa (1978b) (shown in App. A). Example problem 1 in Section V illustrates the method of applying this correction.

The results in Figures 1 to 7 are all presented in terms of the significant wave height at the toe of the structure,  $H_s$ , rather than the deepwater, unrefracted wave height,  $H_0'$ . If it is desired to convert the results of this study to deepwater conditions,  $H_s$  should be multiplied by the shoaling coefficient, given in Appendix C of the SPM (U.S. Army, Corps of Engineers, Coastal Engineering Research Center, 1977), calculated using  $d_s$  and  $T_p$  to obtain an estimate of the deepwater, unrefracted significant wave height.

### III. IRREGULAR WAVE RUNDOWN

Irregular wave rundown is characterized by the 98 percentile rundown,  $Rd_{98}$ , i.e., the rundown depth below the stillwater level which is greater than 98 percent of the wave rundowns. The irregular wave rundown parameter,  $Rd_{98}$  is analogous to the runup parameter,  $R_2$ , since only 2 percent of the rundowns are lower than  $Rd_{98}$ . Figure 8 shows the trend of the relative rundown,  $Rd_{98}/H_s$  as a function of the surf parameter,  $\xi$ , and the approximate upper

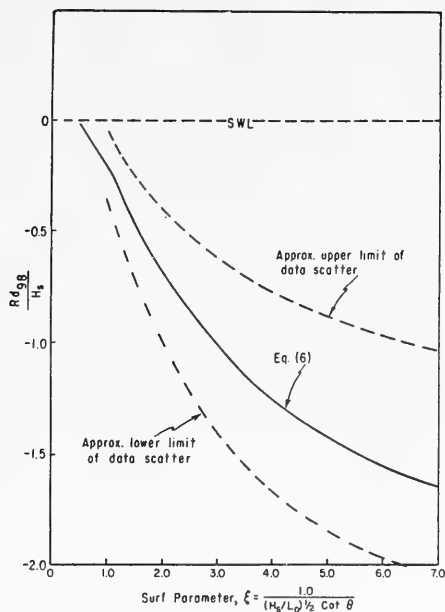


Figure 8.  $Rd_{98}/H_s$  versus the surf parameter.

and lower limits of data scatter about the trend-line curve. The trend-line curve for relative rundown is given by the equation

$$\frac{Rd_{98}}{H_s} = -2.32e^{-2.46/\xi} \quad (6)$$

The absolute value of relative rundown is small for small values of the surf parameter since the plunging waves which dominate these conditions cause considerable wave setup. As the surf parameter increases a standing wave develops against the structure and the relative rundown approaches -1.75, although values occasionally as low as -2.25 were observed. Equation (6) provides a simple way to estimate the approximate lower limit of rundown.

There is no scale-effect correction factor specifically developed for wave rundown, so it is recommended that the correction factor for wave runup be applied to rundown as illustrated in example problem 2 in Section V.

#### IV. APPLICATION OF RESULTS TO ROUGH AND POROUS SLOPES

The results given in this report can be applied to plane, rough- and porous-slope structures, if there is relatively deep water at the toe of the structure (as discussed previously in Sec. II). To apply these results it is necessary to have a reliable estimate of the rough-slope runup correction factor,  $r$ , which is the ratio of wave runup on a rough or porous slope to the



runup on a smooth slope, all other conditions being the same (Stoa, 1978a). Normally,  $r$  is determined in laboratory experiments using monochromatic wave conditions but it appears that  $r$  factors determined in this manner can also be applied to irregular wave conditions (Battjes, 1974). Values of  $r$  for various types of rough and porous slopes are given by Stoa (1979) (shown in App. B).

Often wave runup on rough slopes must be corrected for scale effects and the correction factors are given in Stoa (1979) (shown in App. C). Example problem 3 illustrates how the results presented in this report can be applied to a rough and porous slope and the method of applying the rough-slope scale-effect correction factor.

## V. EXAMPLE PROBLEMS

\*\*\*\*\* EXAMPLE PROBLEM 1 \*\*\*\*\*

This example illustrates the use of the runup equation, Figures 1 to 6, and the recommended method of interpolation between slopes.

GIVEN: A plane, smooth slope of 1 on 2.75 is subjected to irregular wave action. The significant wave height, significant wave period, and water depth at the toe of the structure are 6.0 feet (1.83 meters), 7.0 seconds, and 24.0 feet (7.3 meters), respectively.

FIND:  $\bar{R}$ ,  $R_S$ , and  $R_2$  for the given conditions. Would there be substantial wave overtopping if the freeboard of the structure were 20.0 feet (6.10 meters)?

SOLUTION: Since there is no figure or set of coefficients for the runup equation (eq. 2) for a slope of 1 on 2.75 it is necessary to compute  $R$ ,  $R_S$ , and  $R_2$  for slopes of 1 on 2.50 and 1 on 3.00 and interpolate between them. To start, calculate the period of peak (maximum) energy density,  $T_p$ , using equation (1).

$$T_p = 1.05 T_S = 1.05 (7.0) = 7.35 \text{ seconds}$$

Then compute the steepness parameter,  $H_S/gT_p^2$

$$\frac{H_S}{gT_p^2} = \frac{6.0}{32.2(7.35)^2} = 0.00345$$

Using the above value of steepness in equation (2) with the coefficient given in Table 1 allows the computation of  $R_X/H_S$ . For example, to calculate  $R_2/H_S$  for a 1 on 2.5 slope

$$\frac{R_2}{H_S} = 3.39 + [129.0(0.00345)] + [-16,100(0.00345)^2] = 3.64$$

The above value of  $R_2/H_S$  can be confirmed, using Figure 4. Therefore,

$$R_2 = 3.64(H_S) = 3.64(6.0) = 21.8 \text{ feet (6.64 meters)}$$

The other runup parameters  $R_S$  and  $\bar{R}$  can be calculated in a similar manner, then used for interpolation to give the values of the runup parameters for the 1 on 2.75 slope as shown in Table 2.

Table 2. Values of the runup parameters for example problem 1.

| $\cot \theta$ | $R_2/H_S$ | $R_2$<br>(ft)     | $R_S/H_S$ | $R_S$<br>(ft)     | $\bar{R}/H_S$ | $\bar{R}$<br>(ft) |
|---------------|-----------|-------------------|-----------|-------------------|---------------|-------------------|
| 2.50          | 3.64      | 21.8              | 2.52      | 15.1              | 1.58          | 9.5               |
| 3.00          | 3.49      | 21.0              | 2.43      | 14.6              | 1.56          | 9.4               |
| 2.75          | --        | 21.4 <sup>1</sup> | --        | 14.9 <sup>1</sup> | --            | 9.4 <sup>1</sup>  |

<sup>1</sup>Interpolated value.

The interpolated values in Table 2 should be corrected for scale effects to yield the required answer. The scale correction factor for a slope of 1 on 2.75 is 1.125 (see App. A); therefore,

$$R_2 = 21.4 (1.125) = 24.1 \text{ feet (7.35 meters)}$$

$$R_S = 14.9 (1.125) = 16.8 \text{ feet (5.12 meters)}$$

$$\bar{R} = 9.4 (1.125) = 10.6 \text{ feet (3.28 meters)}$$

A freeboard of 20.0 feet falls between  $R_2$  and  $R_S$ , so the structure crest would not be overtopped frequently, probably by less than 10 percent of the waves. It is, therefore, expected that the volume of overtopping would not be great.

It is difficult to determine how high a smooth structure would have to be to prevent all wave overtopping but a reasonable estimate would be

$$R_{\max} \approx R_2 + H_S$$

where  $R_{\max}$  is the elevation of the maximum runup.

\*\*\*\*\* EXAMPLE PROBLEM 2 \*\*\*\*\*

This example illustrates how to calculate the approximate lower limit of rundown.

GIVEN: A plane, smooth 1 on 2.50 slope is subjected to irregular wave action. The significant wave height, significant wave period, and water depth at the toe of the structure are 7.0 feet (2.13 meters), 8.0 seconds, and 30.0 feet (9.14 meters), respectively.

FIND:  $R_{d98}$  for the above conditions; this is the approximate lower limit of wave rundown.

SOLUTION: The period of peak energy density is

$$T_p = 1.05(T_S) = 1.05 \times 8.0 = 8.40 \text{ seconds}$$

and the surf parameter is

$$\xi = \frac{1}{(H_S/L_0)^{1/2} \cot \theta} = \frac{1}{\{7.0/[32.2 \times (8.4)^2]/2\pi\}(2.5)} = 2.87$$

Using this value of  $\xi$  in equation (6) gives the relative rundown, i.e.,

$$\frac{Rd_{98}}{H_s} = -2.32e^{-2.46/\xi} = -0.99$$

which can be confirmed in Figure 8. Then

$$Rd_{98} = (7.0)(-0.99) = -6.9 \text{ feet } (-2.10 \text{ meters})$$

and using Appendix A to correct this rundown for scale effects gives

$$Rd_{98} \text{ (corrected)} = -6.9(1.128) = -7.8 \text{ feet } (-2.38 \text{ meters})$$

The same scale correction factor used for runoff is used for rundown.

\*\*\*\*\* EXAMPLE PROBLEM 3 \*\*\*\*\*

This example illustrates how the results of tests with irregular waves on smooth slopes can be applied to situations where the structure is rough and porous.

GIVEN: A rubble-mound breakwater is to be built with a slope on the seaward face of 1 on 2 which will be overtopped by wave action only occasionally under the design conditions. The design conditions include a significant wave height, significant wave period, and water depth at the toe of the structure of 15.0 feet (4.57 meters), 12.0 seconds, and 45.0 feet (13.72 meters), respectively. The core of the breakwater will be slightly above the design water level, i.e., a high core breakwater.

FIND: The height at which the breakwater will only occasionally be overtopped during the design conditions.

SOLUTION: The period of peak energy density is

$$T_p = 1.05(T_s) = 1.05 (12.0) = 12.6 \text{ seconds}$$

and the steepness parameter is

$$\frac{H_s}{gT_p^2} = \frac{15.0}{32.2(12.6)^2} = 0.00293$$

Using equation (2) with the coefficients in Table 1 for a plane, smooth slope of 1 on 2 and  $R_2H_s/g$  gives

$$\frac{R_2}{H_s} = 3.2083 + 71.879 (0.00293) = 3.42$$

(this value can be checked in Fig. 3) and

$$R_2 = 3.42(15.0) = 51.3 \text{ feet } (15.64 \text{ meters})$$

The runup reduction factor,  $r$ , for rubble-mound breakwaters with high cores is 0.52 (see App. B) and the scale-effect correction factor is 1.06 (see App. C) so  $R_2$  for the breakwater is

$$R_2 \text{ (breakwater)} = 51.3(0.52) 1.06 = 28.3 \text{ feet (8.63 meters)}$$

$R_S$  and  $\bar{R}$  are found in a similar manner to be

$$R_S \text{ (breakwater)} = 20.0 \text{ feet (6.10 meters)}$$

$$\bar{R} \text{ (breakwater)} = 12.2 \text{ feet (3.72 meters)}$$

These calculations indicate that if the freeboard were 28.3 feet only 2 percent of the waves with a  $H_S = 15$  feet and  $T_S = 12$  seconds spectrum would overtop the structure while a freeboard of 12.2 feet would allow about half the waves to overtop. A freeboard equal to  $R_S$ , i.e., 20 feet, will satisfy the condition of only occasional wave overtopping since about 13 percent of the waves would be expected to overtop the breakwater.

\*\*\*\*\*

## VI. SUMMARY

Equations and curves are presented for computing three runup parameters and one rundown parameter for plane, smooth slopes exposed to irregular wave conditions where  $d_S/H_S > 3$ . These parameters are  $R_2$ , the elevation exceeded by only 2 percent of the runups;  $R_S$ , the average runup of the highest one-third of the wave runups;  $R$ , the mean runup of all the runups; and  $R_{d98}$ , the depth below the stillwater level which is just greater than 98 percent of the rundown. Example problem 1 illustrates the use of equation (2) in computing the rundowns, parameters, and the method of interpolation for runup on slopes not specifically covered in this report. Example problem 2 illustrates the method of computing rundown. Example 3 illustrates how the study results for smooth slopes can be applied to rough and porous slopes, in this case to compute the desired freeboard for a rubble-mound breakwater.

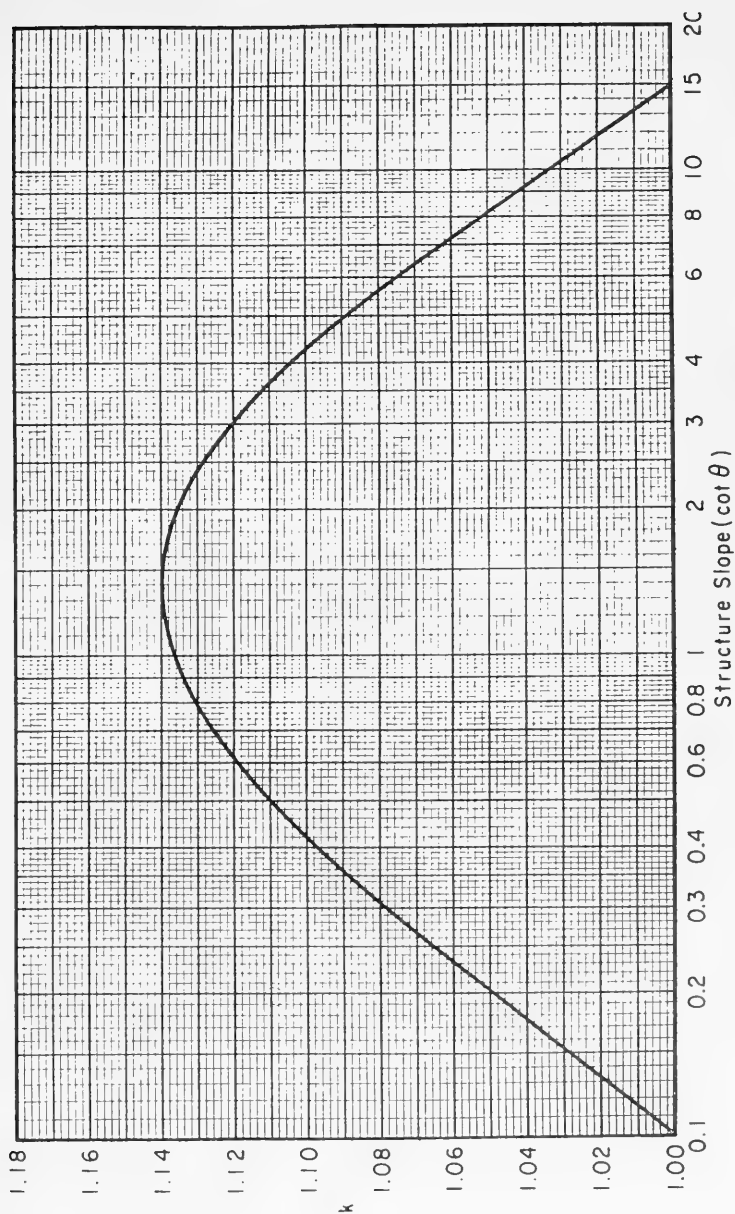
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# APPENDIX A

RUNUP SCALE-EFFECT CORRECTION FACTOR,  $k$ , FOR SMOOTH SLOPES (Stoa, 1978a)

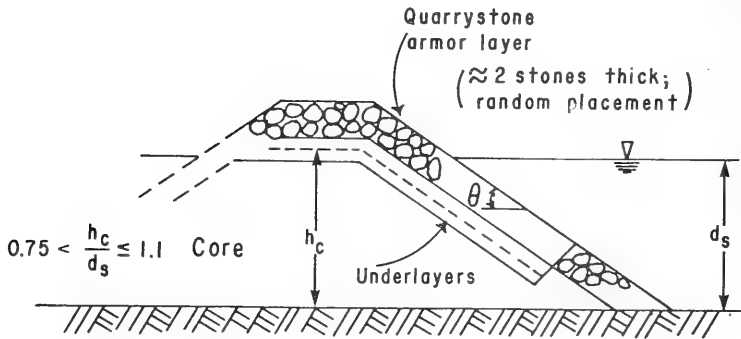


# APPENDIX B

## RUNUP REDUCTION FACTOR, $r$ , FOR VARIOUS TYPES OF ROUGH AND POROUS STRUCTURES (Stoa, 1979)

### I. VALUE OF $r$ FOR QUARRYSTONE RUBBLE-MOUND STRUCTURE (HIGH CORE)

$$r = 0.52$$





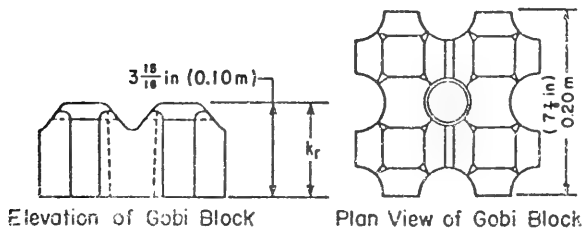
## II. VALUES OF $r$ FOR CONCRETE ARMOR UNITS

### 1. Embankment.

#### a. Gobi Blocks.

$$r \approx 0.93 \text{ for } H'_O/k_r \text{ or } H/k_r \approx 6$$

(use  $H'_O$  when  $d_s/H'_O \geq 3$  and  $H$  when  $d_s/H'_O < 3$ )



#### b. Stepped Slopes.



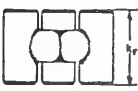
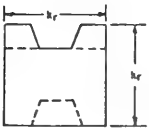
Values of  $r$  for stepped slopes.

| Type of step    | Slope (cot $\theta$ ) | $r^1$ |
|-----------------|-----------------------|-------|
| Vertical risers | 1.5                   | 0.75  |
|                 | 2.0                   | 0.75  |
|                 | 3.0                   | 0.70  |
| Rounded edges   | 3.0                   | 0.86  |

<sup>1</sup> $1 < H'_O/k_r \leq 12$  where  $k_r$  is the height of the riser.

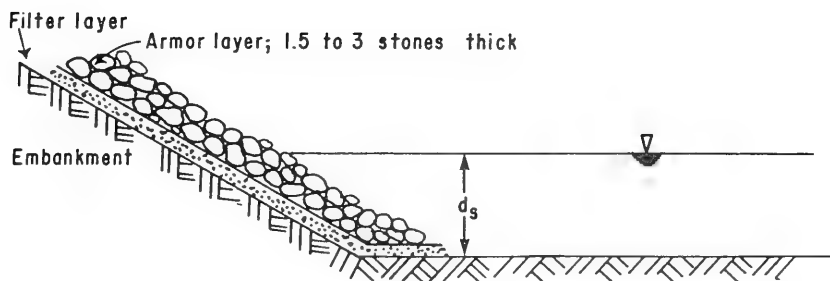
2. Embankment and Rubble Mound.

Values of  $r$  for concrete armor units.

| Armor unit and placement method                                      | Length dimension, $k_r$   | Armor-layer thickness (No. of units) | Values of $r$ | Slopes ( $\cot \theta$ ) |
|--|---|--------------------------------------|---------------|--------------------------|
| Tetrapod<br>Random<br><br>Uniform                                    |  | 2                                    | 0.45          | 1.3 to 3.0               |
|  |   | 2                                    | 0.51          | 1.3 to 3.0               |
| Quadripod<br>Random<br><br>Uniform                                   |  | 2                                    | 0.51          | 1.3 to 3.0               |
|  |   | 2                                    | 0.51          | 1.3 to 3.0               |
| Tribar<br>Random<br><br>Uniform                                      |  | 2                                    | 0.45          | 1.3 to 3.0               |
|  |   | 1                                    | 0.50          | 1.3 to 3.0               |
| Modified cube<br>Random<br><br>Uniform<br><br>Uniform<br><br>Uniform |  | 2                                    | 0.48          | 1.3 to 3.0               |
|  |   | 1                                    | 0.62          | 1.5                      |
|  |   | 1                                    | 0.73          | 2.0                      |
|  |   | 1                                    | 0.55          | 3.0                      |

### III. VALUES OF $r$ FOR QUARRYSTONE EMBANKMENT

| Slope ( $\cot \theta$ ) | $H/k_p$ | $r$  |
|-------------------------|---------|------|
| 1.5                     | 3 to 4  | 0.60 |
| 2.5                     | 3 to 4  | 0.63 |
| 3.5                     | 3 to 4  | 0.60 |
| 5.0                     | 3       | 0.60 |
| 5.0                     | 4       | 0.68 |
| 5.0                     | 5       | 0.72 |



# APPENDIX C

RUNUP SCALE CORRECTION FACTOR,  $k$ , FOR VARIOUS  
TYPES OF ROUGH AND POROUS STRUCTURES (Stoa, 1979)

| Structure Type                                     | $k$  |
|--|------|
| Quarrrystone, rubble-mound breakwater              | 1.06 |
| Quarrrystone, riprap revetment                     | 1.00 |
| Concrete armor units, rubble mound<br>or revetment | 1.03 |

|  |  |
|--|--|
| <p>Ahrens, John P.<br/>Irregular wave runup on smooth slopes / by John P. Ahrens.--Fort Belvoir, Va. : U.S. Army Coastal Engineering Research Center ; Springfield, Va. : available from NTIS, 1981.<br/>[26] p. : ill. ; 28 cm.--(Coastal engineering technical aid ; no. 81-17)<br/>Cover title.<br/>"December 1981."<br/>Bibliography: p. 19.<br/>The results of several laboratory studies have been used to develop a method to estimate the wave runup and rundown on plane, smooth slopes caused by irregular wave action. Curves and equations are presented which can be used to compute the 2-percent runup, significant runup, mean runup, and approximate lower limit of rundown. A procedure is suggested for adapting the smooth-slope results to wave runup on rough and porous slopes. Example problems illustrate the use of the material presented.<br/>1. Wave runup. 2. Water waves. I. Title. II. Series.<br/>TC203 no. 81-17 .U581ta 627</p> | <p>Ahrens, John P.<br/>Irregular wave runup on smooth slopes / by John P. Ahrens.--Fort Belvoir, Va. : U.S. Army Coastal Engineering Research Center ; Springfield, Va. : available from NTIS, 1981.<br/>[26] p. : ill. ; 28 cm.--(Coastal engineering technical aid ; no. 81-17)<br/>Cover title.<br/>"December 1981."<br/>Bibliography: p. 19.<br/>The results of several laboratory studies have been used to develop a method to estimate the wave runup and rundown on plane, smooth slopes caused by irregular wave action. Curves and equations are presented which can be used to compute the 2-percent runup, significant runup, mean runup, and approximate lower limit of rundown. A procedure is suggested for adapting the smooth-slope results to wave runup on rough and porous slopes. Example problems illustrate the use of the material presented.<br/>1. Wave runup. 2. Water waves. I. Title. II. Series.<br/>TC203 no. 81-17 .U581ta 627</p> |
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